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A Comment on Two Current Adhesive Lap Joint Theories

William C. Carpenter*
University of South Florida, Tampa, Fla.

Introduction

SOME 30 years ago, Goland and Reissner[1] presented one of the most significant papers on bonded connections. Ojalvo and Eidinoff[2] recently pointed out that Goland and Reissner's theory was derived using an incomplete shear strain-displacement relationship. They modified Goland and Reissner's basic equations and presented examples which indicated the difference between the original and the revised theory. This Comment points out that both Goland and Reissner and Ojalvo and Eidinoff, in deriving the differential equations of stress in the adhesive, made arbitrary choices of which equations to combine in order to obtain their resulting differential equations. The derivations of this paper yield differential equations which reduce to those of Goland and Reissner and Ojalvo and Eidinoff upon selection of appropriate values of arbitrary coefficients.

Both the theories of lap joint behavior of Goland and Reissner and of Ojalvo and Eidinoff examine equilibrium of a unit width differential element of an adherend-adhesive configuration as shown in Fig. 1. Table 1 summarizes the equilibrium equations and the force-displacement or stress-displacement equations for the configuration. The next two sections briefly detail the development of the differential equations for the shear and normal stress in the adhesive.

Table 1 Equations for the adhesive element[a]

Type of equation	Adherend 1		Adherend 2		Adhesive		Composite element
Vertical force equilibrium	$Q_1' - \sigma_1 = 0$	(1)	$Q_2' + \sigma_2 = 0$	(6)	$Q_a' = \sigma_2 - \sigma_1$	(11)	$Q_1' + Q_2' + Q_a' = 0$ (16)
Moment equilibrium	$M_1' + Q_1 - (t/2)\tau_1 = 0$	(2)	$M_2' + Q_2 - (t/2)\tau_2 = 0$	(7)	$Q_a = \tau_0 h$	(12)[b]	Not required
Horizontal force equilibrium	$N_1' - \tau_1 = 0$	(3)	$N_2' + \tau_2 = 0$	(8)	$\tau_1 = \tau_2$	(13)	Not required
Force-displacement or	$u_1' = \frac{1}{Et}\left(\frac{6M_1}{t} + N_1\right)$	(4)	$u_2' = \frac{1}{Et}\left(\frac{-6M_2}{t} + N_2\right)$	(9)	$\tau_a = G_a\left[\frac{u_1 - u_2}{h} + \alpha_1\left(\frac{w_1' + w_2'}{2}\right) + \alpha_2 (z/h)(w_1' - w_2')\right]$	(14)[c]	Not applicable
Stress displacement relationships	$w_1'' = 12M_1/Et^3$	(5)	$w_2'' = 12M_2/Et^3$	(10)	$\sigma_a = E_a/h(w_1 - w_2)$	(15)[d]	

[a] Typical units for Q_i and N_i are lb/(in. width); for M_i, lb-in./(in. width); for σ_i and τ_i, lb/in.^{2}. [b] $\tau_0 = (\tau_1 + \tau_2)/2$. [c] α_1 and α_2 are constants which depend upon the shear stress-displacement equation used for the adhesive. [d] Assuming Poisson's ratio for the adhesive to be zero.

Shear Stress

From Eq. (14)

$$\tau_0 = G_a\left[\frac{u_1 - u_2}{h} + \alpha_1\left(\frac{w_1' + w_2'}{2}\right)\right] \tag{17}$$

where

$$\tau_0 = (\tau_1 + \tau_2)/2 \tag{18}$$

Differentiating Eq. (17) and entering Eqs. (4), (9), (5), and (10) into the resulting equation gives

$$\tau_0' = \frac{G_a}{Eth}\left[N_1 - N_2 + \frac{6(1 + \alpha_1\beta)}{t}(M_1 + M_2)\right] \tag{19}$$

where

$$\beta = h/t \tag{20}$$

Differentiating Eq. (19) and entering in Eqs. (3), (8), (2), and (7) gives

$$\tau_0'' = \frac{G_a}{Eth}\left[(8 + 6\alpha_1\beta)\tau_0 - \frac{6(1 + \alpha_1\beta)}{t}(Q_1 + Q_2)\right] \tag{21}$$

Differentiation of Eq. (21) gives

$$\tau_0''' = \frac{G_a}{Eth}\left[(8 + 6\alpha_1\beta)\tau_0' - \frac{6(1 + \alpha_1\beta)}{t}(Q_1' + Q_2')\right] \tag{22}$$

Now the moment equilibrium equation for the adhesive, Eq. (12), gives

$$0 = -Q_a' + h\tau_0' \tag{23}$$

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*Associate Professor, Department of Structures, Materials & Fluids.

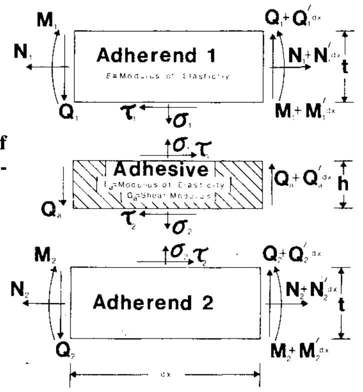

Fig. 1 Differential element of an adherend-adhesive configuration.

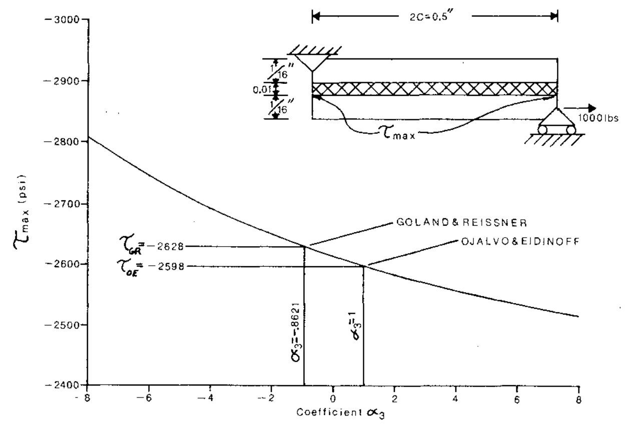

Fig. 2 Variation of maximum shear stress with coefficient α_3 for $\alpha_1 = 1$.

or

$$0=\alpha_3\left[\frac{-6}{t}(1+\alpha_1\beta)Q_a'+6(1+\alpha_1\beta)\beta\tau_0'\right] \tag{24}$$

where α_3 is an arbitrary constant. When $\sigma_1=\sigma_2$ (the assumption of both Goland and Reissner and Ojalvo and Eidinoff) the vertical force equilibrium equation for the adhesive, Eq. (11) gives

$$0=Q_a' \tag{25}$$

or

$$0=\alpha_4\left[\frac{-6(1+\alpha_1\beta)}{t}Q_a'\right] \tag{26}$$

where α_4 is an arbitrary constant.

Adding Eqs. (22), (24), and (26) gives

$$\tau_0'''=\frac{G_a}{Eth}\left[(8+6\alpha_1\beta+6\alpha_3\beta+6\alpha_1\alpha_3\beta^2)\tau_0' \times\frac{-6(1+\alpha_1\beta)}{t}(Q_1'+Q_2'+\alpha_3Q_a'+\alpha_4Q_a')\right] \tag{27}$$

If

$$\alpha_3+\alpha_4=1 \tag{28}$$

then from Eqs. (27), (28), and (16)

$$\tau_0'''-(\lambda^2/c^2)\theta\tau_0'=0 \tag{29}$$

where

$$\lambda^2=G_ac^2/Eth \tag{30}$$

$$\theta=8+6\alpha_1\beta+6\alpha_3\beta+6\alpha_1\alpha_3\beta^2 \tag{31}$$

and $2c$ is the lap length of the joint in question.

It can be noted here that the differential equation for τ_0, Eq. (29), depends upon α_1 and α_3 but is independent of α_2. In other words, the nature of the differential equation for τ_0 is dependent upon the constant terms in the shear stress expression, Eq. (14), but is independent of the linear varying term in that equation. In the differential equation for τ_0, the constant α_3 can be chosen arbitrarily as α_4 can be varied as required to satisfy Eq. (28). An example problem is later solved to show the dependence of the shear stress τ_0 on the constant α_3. It should be noted that $\alpha_1=\alpha_3=1$ gives the differential equation of Ojalvo and Eidinoff and that either $\alpha_1=\alpha_3=0$ (as was pointed out by Ojalvo and Eidinoff) or $\alpha_1=1$ and $\alpha_3=-1/(1+\beta)$ gives the differential equation of Goland and Reissner. Thus both the differential equation for shear stress of Goland and Reissner and of Ojalvo and Eidinoff can be derived using a complete shear stress-displacement equation ($\alpha_1=1$) together with an appropriate value for α_3.

Normal Stress

Differentiating Eq. (15) thrice and entering in Eqs. (5) and (10) gives

$$\sigma_a'''=\frac{E_a}{h}\left(\frac{12M_1'}{Et^3}\ \frac{-12M_2'}{Et^3}\right) \tag{32}$$

Entering Eqs. (2) and (7) into Eq. (32) gives

$$\sigma_a'''=\frac{12E_a}{Et^3h}\left([\tau_1-\tau_2]\frac{t}{2}-Q_1+Q_2\right) \tag{33}$$

Combining Eqs. (14), (15), and (33) gives

$$\sigma_a'''=\frac{12E_a}{Et^3h}\left(\alpha_2\frac{G_ah}{E_a}\sigma_a'\frac{t}{2}-Q_1+Q_2\right) \tag{34}$$

When $\sigma_1=\sigma_2=\sigma_a$, differentiating Eq. (34) and entering in Eqs. (1) and (6) gives

$$\sigma_a^{iv}-\frac{6\alpha_2\lambda^2\beta}{c^2}\sigma_a''+\frac{\rho^2}{c^4}\sigma_a=0 \tag{35}$$

where

$$\rho^2=24E_ac^4/Et^3h \tag{36}$$

It can be noted here that the differential equation for normal stress, Eq. (35), depends upon α_2 and thus upon the linear varying term in the shear stress expression, Eq. (14), but is independent of α_1 and thus is independent of the constant term in that equation. For $\alpha_2=0$, Eq. (35) gives the differential equation of Goland and Reissner, while $\alpha_2=1$ gives the differential equation of Ojalvo and Eidinoff.

Example

Figure 2 shows a unit width lap joint which was investigated to determine the effect of the arbitrary constant α_3 on the solution for shear stress from Eq. (29). This example has

$$(G/h)_a=10^7\text{lb/in.}^3,\quad (E/h)_a=2.8\times10^7\text{lb/in.}^3,\quad E_{\text{plate}}=16.5\times10^6\text{lb/in.}^2,\ \text{and}\ \beta=0.16 \tag{37}$$

and thus from Eq. (31) when $\alpha_1=1$,

$$\theta=8.96+1.1136\alpha_3 \tag{38}$$

Figure 2 also shows the variation of the maximum shear stress with variation of the constant α_3. When $\alpha_3=1$, θ reduces to that given by Ojalvo and Eidinoff and when $\alpha_3=-0.8621$ to that given by Goland and Reissner.

Conclusion

The differential equation for normal stress in the adhesive of a lap joint as proposed by Ojalvo and Eidinoff is an improvement over that proposed by Goland and Reissner in that the differential equation of normal stress by Goland and Reissner is based upon an incomplete shear stress-displacement relationship, while that of Ojalvo and Eidinoff is based on the complete relationship. However, the differential equation for shear stress in the adhesive, using a complete shear stress-displacement relationship and assumptions common to the theories of both Goland and Reissner and Ojalvo and Eidinoff is not unique in that that it contains an arbitrary constant. Examination of a wide range of values of that constant on a sample problem indicates that shear stress values vary significantly with the choice of the constant. Different choices of the constant give either the differential equation of Goland and Reissner or of Ojalvo and Eidinoff.

References

[1] Goland, M. and Reissner, E., "The Stresses in Cemented Joints," *Journal of Applied Mechanics,* Vol. 11, March 1944, pp. A17-A27.

[2] Ojalvo, I.U. and Eidinoff, H.L., "Bond Thickness Effects Upon Stresses in Single-Lap Adhesive Joints," *AIAA Journal* Vol. 16, March 1978, pp. 204-211.

J80-067

Reply by Authors to W.C. Carpenter

I. U. Ojalvo* and H. L. Eidinoff†
Grumman Aerospace Corp., Bethpage, N.Y.

THE purpose of this Reply is to refute the main point of Carpenter's Comment, i.e., that there are arbitrary coefficients in our lap joint adhesive theory as presented in Ref 1. It is our contention that there is a basic error in Carpenter's theory associated with the satisfaction of transverse deflection compatibility at the bond/adherend interface. The following comments contain a development of this position.

The first two arbitrary constants introduced by Carpenter in Eq. (14) of Table 1 contain no explanation other than a footnote which states "α_1 and α_2 are constants which depend upon the shear stress-displacement equation for the adhesive." We shall demonstrate here that α_1 and α_2 are not at all arbitrary, but must be fixed values if the adhesive displacements u_a and w_a are to be compatible with the adherend displacements $\bar{u}_1$ and w_1 at their contacting surfaces ($z=\pm h/2$, $|x|<c$). These symbols are defined in Figs. 1 and 2. It should be noted that u_1 (undefined by Carpenter) is called $\bar{u}_1$ here and in Ref. 1.

The theory of Ref. 1 was based upon a number of basic assumptions. The assumptions regarding the adhesive which are required for our present Reply are: 1) the adhesive strain-displacement equation is linear, 2) the adhesive material is linearly elastic, and 3) the deflections vary linearly through the bond thickness. Assumptions 1 and 2 were implicity stated as Eqs. (11) and (12) of Ref. 1, while the third assumption was explicitly stated in that work.

For compatability of adhesive and adherend displacements at $z=\pm h/2$, assumption 3 requires that

$$u_a=(\bar{u}_1+\bar{u}_2)/2+(z/h)(\bar{u}_1-\bar{u}_2) \tag{1}$$

and

$$w_a=(w_1+w_2)/2+(z/h)(w_1-w_2) \tag{2}$$

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*Structural Mechanics Group Leader.
†Structural Analysis Engineer.

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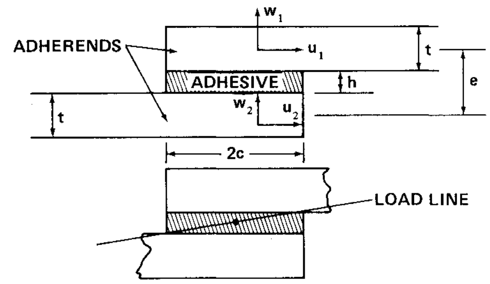
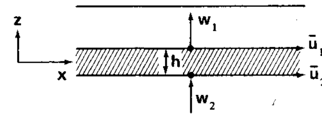


Fig. 1 Unloaded joint—eccentricity, $t+h$.



$$u_a=\frac{\bar{u}_1+\bar{u}_2}{2}+\frac{z}{h}(\bar{u}_1-\bar{u}_2)$$

$$w_a=\frac{w_1+w_2}{2}+\frac{z}{h}(w_1-w_2)$$

Fig. 2 Adhesive displacements—linear variation through the thickness.

which are identical to Eqs. (10a and b) of Ref. 1.

Combining assumption 1 with Eqs. (1) and (2) yields

$$\gamma_a=\frac{\partial u_a}{\partial z}+\frac{\partial w_a}{\partial x}=\frac{\bar{u}_1-\bar{u}_2}{h}+\frac{w_1'+w_2'}{z}+\frac{z}{h}(w_1'-w_2') \tag{3}$$

Combination of Eq. (3) with assumption 2 yields

$$\tau_a=G\gamma_a=G\left(\frac{\bar{u}_1-\bar{u}_2}{h}+\frac{w_1'+w_2'}{z}+\frac{z}{h}(w_1'-w_2')\right) \tag{4}$$

which differs from Carpenter's Eq. (14) in that α_1 and α_2 do not appear. It is seen that the only way in which his Eq. (14) will agree with Eq. (4) above is if α_1 and α_2 are both unity, in which case they are fixed and not arbitrary.

The two remaining arbitrary constants in Carpenter's "Comment," α_3 and α_4, appear in a somewhat mysterious fashion in his Eqs. (24) and (26). These terms arbitrarily multiply terms which are identities and thus zero [see his Eqs. (23) and (25)] and are appended to Eq. (22) to become his Eq. (27). He then proceeds to arbitrarily introduce Eq. (28) with no explanation. Presumably, he has done this to reconcile the fact that our technical theory contains an inconsistency. It is our contention that we have admitted this inconsistency in Ref. 1 in our discussion following our Eq. (27c). Such inconsistencies arise from the fact that a displacement field was assumed (for the adhesive) which did not satisfy the adhesive equilibrium equations. The justification for our assumption was that 1) the transverse shear carried by the adhesive was negligible compared to that carried by the adherends and 2) the displacement field chosen was the simplest consistent one possible. We feel Carpenter should have offered some rationale for selecting α_3 and α_4 such as a strain energy (U) formulation in which perhaps U is minimized relative to variations in α_3 and α_4.

In conclusion, it is our position that Carpenter is in error in stating that α_1 and α_2 are arbitrary since deflection compatibility is violated if these coefficients are not equal to unity, and that he has introduced arbitrary constants, α_3 and α_4, which should be justified on some physical grounds.

References

[1] Ojalvo, I.U. and Eidinoff, H.L., "Bond Thickness Effects upon Stresses In Single-Lap Adhesive Joints," *AIAA Journal,* Vol. 16, ~~March~~ 1978, pp. 204-211.

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